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Subject: Mathematics

Title of activity: Sharing the cake

Level: 1st and 2nd year of ESO (Ages: 12-14)

Justification: Maths usually requires a certain level of abstraction. They are frequently complicated for adults who have their minds developped so for kids in their early teens, sometimes they become unreachable. This is the reason why we try to acquire and discover the mathematical concepts with objects they can touch. The handwork must be a previous step to the abstract reasoning.

We also try to provide experiences to remember and, of course, to have fun.

Objectives:

- Concept of fraction. Parts we take from a total.

- Addition of fractions with the same denominator

- Similar/ equivalent fractions: Concept, algorythm to verify if two fractions are equivalent, methods (amplification and reduction) to obtain similar fractions.

- Addition of fractions with the different denominator

Description of the activity:

We work the concept of similar/equivalent fractions and the addition or subtraction of fractions with different denominator by bringing to class under any false pretense (a birthday, to celebrate a good result in an exam, as an offering of peace after a row,...) two rectangular cakes: one of chocolate and the other of lemon. First, we work the concept of fraction and its components (numerator & denominator), its "complementary" (the fraction that remains to complete the unit and the algorythm of addition with the same denominator. At the same time, we are doing a bit of theatre with two other models of cake made with scourer and wrapped in brown for the chocolate and in transparent for the lemon. The reason for this is not to be touching with our hands the food we are going to eat afterwards. As we are handling the models we ask students to repeat the process graphically in the design we provide and to answer the questions.

Then we try to add fractions with different denominator with the same process until we realize it is impossible doing it without thinking. The key is when they try to establish the denominator as the pieces ofchocolate are not of the same width as those of lemon. Thus, we must find a way to have all the pieces of the same. As they are doing that we, that have it prepared, turned round the scoures. On one side we have $\frac{1}{3}$ (one piece of chocolate cake out of the three pieces our cake was divided into) and turning around we have $\frac{5}{15}$. Obviously, they represent the same amount of cake because they use the same space but,at the same time, fractions are different. At this moment we introduce the concept of equivalent fractions, how to obtain similar fractions and the method to check if two fractions are equivalent.

The addition/subtraction of fractions is now immediate but, to finish, we analize the mathematic process. We conclude discovering the method of addition of fractions which consists in obtaining the denominator multiplying the denominators first and the numerators multiplying in cross. We compare this method with the tradicional one of obtaining the Least Common Multiple. In this last method we work with smaller equivalent fractions but in the first one procedures are much easier.

Document students work: In the next page

Introduction to fractions. Sharing the cakes

As I'm very pleased with your hard work, I have decided to have a little party. That's why I have brought two cakes; the first one has been cut into three pieces and the second one into five pieces. The only problem is, as you can imagine, we have to solve some mathematical questions before we eat it.

First of all, I want you to draw in your notebook three equal 15 x 10 rectangles if you have big squares or 30x15 rectangles if you have reasonable squares. That means three rectangles with 15 (or 30) squares long and 10 (or 15) squares wide. We are going to label the first rectangle as "Cake 1: chocolate", the second rectangle as "Cake 2: lemon" and the third one as "Addition".

Cake 1: chocolate. Draw in the first rectangle what I have done: the cake divided in three equal parts. Do it vertically (attending to the 15 or 30 squares)

a) I am taking out one piece of cake 1 and I put it on a new dish. Draw this in the first rectangle; can we express that amount as a fraction?
b) What number do we write as the numerator and which one as the denominator?
c) Can you express the amount of chocolate cake that remains on the first dish as a fraction?
d) Now, I´m putting back to the first dish what I had taken aside. Can you express with a mathematical operation what I have done? $\frac{\quad}{\quad} + \frac{\quad}{\quad} = \frac{\quad}{\quad}$
e) How do you add fractions with the same denominator?

Cake 2: Lemon. Draw in the second rectangle what I have done: the cake divided in five equal parts. Do it vertically (attending to the 15 or 30 squares)

a) I am taking out two pieces of cake B and I put them on a new dish. Draw this in the second rectangle; can we express that amount as a fraction?
b) What number do we write as the numerator and which one as the denominator?
c) Can you express the amount of lemon cake that remains on the first dish as a fraction?
d) Now, I´m putting back to the first dish what I had taken aside. Can you express with a mathematical operation what I have done? $\frac{\quad}{\quad} + \frac{\quad}{\quad} = \frac{\quad}{\quad}$
e) How do you add fractions with the same denominator?

Addition

If I gave someone the three pieces of cake (one from cake A and two from cake B).

Don't be afraid to write your answers, we'll have time to change it later if don't agree

a) Can you express that amount as a fraction?
b) What number do we write in the numerator?
c) What number do we write in the denominator?
d) Are all the pieces I have given the person of the same size?
   Which pieces are bigger: those of the lemon cake or those of the chocolate cake?
e) This is the time for you to change the previous answers if you think so

Time to think: Shouldn't we try to cut the cake so that the pieces are of the same size?

Remember that in a fraction the denominator is the number of equal parts the unit is divided into

What do we have to do to have both cakes (the first one divided in three equal parts and the second one divided in five) divided in the same number of pieces?

Meanwhile, I cut the cakes as we had agreed, do it yourself in the three rectangles (now all of them must be divided in the same number of parts and all of the parts must be equal).

We are going to repeat the experiment

Cake A: Chocolate. Look at the first rectangle.

In how many pieces do we have cake A now?
How many pieces of cake A are we taking away to take away the same amount I did before?
Express as a fraction the amount of the lemon cake we have taken

Cake B: Lemon. Look at the second rectangle.

In how many pieces do we have cake B now?
How many pieces of cake A are we taking away to take away the same amount I did before?

Express as a fraction the amount of the lemon cake we have taken

Rectangle 3. Addition. I put all the pieces, those from cake A and those from cake B, in a new tray

Do the same in your third rectangle.

In how many pieces is the third rectangle divided into?
How many pieces of cake am I giving to that person now?
Can you express this amount of cake as a fraction? Write the whole process and the answer

$\frac{1}{3} + \frac{2}{5} = - + - = -$

Conclusion

a) How have we obtained the denominator when adding two fractions with different denominators?
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b) How have we obtained the numerator when adding two fractions with different denominators?

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