Author: Hilario Barrera Rodríguez, Class teacher

School, country: IES Valsequillo , Gran Canaria, Canary Islands, Spain

Subject: Mathematics

Title of activity: Running around the school yard

Level: 1st and 2nd year of ESO (Ages: 12-14)

Justification: Maths usually requires a certain level of abstraction. They are frequently complicated for adults who have their minds developped so for kids in their early teens, sometimes they become unreachable. This is the reason why we try to acquire and discover the mathematical concepts with experiences of their everyday life.

The idea is to provide experiences to remember and, of course, to have fun.

Objectives:

- Rounding numbers using logical reasoning.

- Decimal and sexagesimal time systems. How to change from one to the other and the other way round.

- Average or mean as a statistical parameter.

- Concept of multiple and find the first eighteen multiples of a number.

- Realization in a practical case of the convenience of learning and using the optimal method to obtain the Least Common Multiple.

Description of the activity: We set students in groups of three and we inform them that they will have to run around the school football pitch. One of them will have to walk, another to run slowly and the last one to run fast. In theory, they will have to do it non-stop keeping the same rythm until they coincide again in the starting line. In the practice, as the real experiment would take too long, they will only have to do it three times timing the runs in minutes (if necessary), seconds and hundreths of seconds. The following day, at the classroom, we develop the three data of each student in an only number (for each) and we guide them in the discovery of the L.C.M.

Document students work: In the next page

RUNNING AROUND THE FOOTBALL PITCH

1. Data collection

Team number	First lap	Second lap	Third lap	
Walker:				
In minutes and seconds				
In seconds				Time :___
Slow runner:				
In minutes and seconds				
In seconds				Time : ____
Fast runner:				
In minutes and seconds				
In seconds				Time :_____

2. When each "runner" crosses the starting line.

	1	2	3	4	5	6	7	8	9
Walker									
S. runner									
F. runner									

	10	11	12	13	14	15	16	17	18
Walker									
S. runner									
F. runner									

3. When do the three students coincide at the finish line?

4. Express the time the three students coincide at the finish line in hours, minutes and seconds. It is possible that you will have to use days as well.

5. How many times will each student cross the finish line before they coincide?

Document we use as support

Running around the school yard

Objective: Three people are going to start running around the school yard at the same time and from exactly the same place but with different speeds: the first one will walk, the second one will run slowly and the third one will run fast. We want to know when the three people will coincide again at the starting line and how many laps will each of them have done.

Situation: As it would be too long, too tiring and too hard to keep the same rhythm for such a long time, we are only going to run (each person) three times and we solve mathematically the problem.

1. - Timing.

Students will have to register their times in a table like this in their notebooks. It would be fantastic if the timing is in minutes, seconds and hundredths of seconds. If that's not possible in minutes, seconds and tenths of seconds will be OK.

Names	First lap	Second lap	Third lap
Walker:			
Slow runner:			
Fast runner:			

2.- Rounding to seconds.

Examples:

a.- Let's suppose the time of the walker has been 1'23"34. Is it closer to 1'23" or closer to 1'24"?

Note: Remember that in a watch the hundredths of second go from 0 to 99. Minutes and seconds finish in 59.

Walker: 1'23"34 →

b.- Let's suppose the time of the slow runner has been 48"63. Is it closer to 48" or closer to 49"? If we wrote from 48"00 to 49"00 on a straight line, would it be closer to the left or the right end?

----|------|----|----|----|----|----|----|----|----|----|----|----|----|--|-|----|----|----|----|----|----|----|----|----

48"00 05 10 15 20 25 30 35 40 45 50 55 60 65 70 75 80 85 90 95 49"00

Slow runner: 48''63→

c.- Let's suppose the time of the runner has been 27"88. Is it closer to 27" or closer to 28"?

Fast runner: 27"88→

Exercise: Round the data in your table to minutes and seconds

3.- Expressing the time only in seconds.

Example:

From the previous step we might have established 2' 15" as the time of the walker.

a. - How many seconds are there in the two minutes?

b. - How many seconds are there in the two minutes and 15''?

2’ 15’’ are _____ seconds

Exercise: Change all the data in your table to only seconds.

4. - What time do we take for each runner?

As our speed is not always exactly the same, we have done the running three times to be more accurate (precise).

Example:

If from the previous step we have established that one of the walkers has taken 90, 91 and 93 to complete the different rounds, how can we reduce these three marks to only one? The final qualification in any subject might help to do it.

Average time=

Exercise: Obtain the average time of the three students (the walker, the slow runner and the fast runner)

5. - Round the average times of three students to seconds (eliminate the possible decimals)

6. - When would the runners get to the finish line in each round?

Imagine that the walker needs 63 seconds to complete a lap, how long will he need to complete two laps? And three laps?

What mathematical operation have you used?

Use the following table to write down the time each runner crosses the finish line in each round.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
Walker																		
Slow runner																		
Fast runner																		

7. - Did you find when the three students coincide on the finish line?

Or would you need to carry on in the same way for 19, 20, 21,… laps?

8. - Either if you reach your conclusion or not, what you have been trying to obtain is not other that “the lowest common multiple” mathematical procedure.

There is an easier way to do it. Pay attention

Example:

Let’s assume the average times are 80, 56 and 32 seconds respectively. We factorize the different numbers

80	2
40	2
20	2
10	2
5	5
1	

56	2
28	2
14	2
7	7
1	

32	2
16	2
8	2
4	2
2	2
1	

$$\left.\begin{array}{l} 80 = 2^4 \cdot 5 \\ 56 = 2^3 \cdot 7 \\ 32 = 2^5 \end{array}\right\}$$ L.C.M. (80,56;32) = $2^5 \cdot 5 \cdot 7 = 32 \cdot 35 = 1120$ seconds

This is the process we have followed:

1) First we take the factors common to the three: 2^4 ; 2^3 and 2^5

2) Which exponent is bigger? 5 so we write L.C.M. (80;56;32) = $2^5 \cdot$

3) Now as they are no other common factor we write the non common factors (if one of them was repeated only in two of the numbers but not in the third one, we will write it with the biggest exponent.

That's why we had L.C.M. (80; 56; 32) = $2^5 \cdot 5 \cdot 7$

Exercise: Obtain the time using the last method the three students will coincide on the finish line.

9. - Change the last result to minutes and seconds dividing by 60 because there are 60 seconds in a minute.

10. - Divide the minutes you obtained in last exercise by 60 in order to be able to say the number of hours, minutes and seconds the three students will coincide

11- In this moment you know how long each person takes to go around the football pitch and you also know how long it takes until the three students coincide at the finish line. Obtain the number of laps each student do until they coincide at the finish line

12. - The time you found the three students will need to coincide again at the finish line, is it bigger or smaller than the times the students need to complete a lap?